From a theory of knowledge to a theory of decision and action

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Outline of the lecture

- Motivation for the approach
- The basic framework: Measurable systems
- The Type-Indeterminacy model
- Strategic decision-making

Motivating the approach

1. Epistemological foundations
2. From knowledge to action
3. Observational analogies

The epistemological challenge of QM:

Quoting Dirac "there is a fineness of our power of observation and the smallness of the accompanying disturbance - a limit which is inherent in the nature of things and can never be surpassed by improved technique or increased skill on the part of the observer" "When the limiting disturbance is not negligeable, then the object is "small" and requires a new theory to deal with it. **As a consequence we must revise our idea about causality**" "

"Even if the state of things cannot be (first hand) objectivised, - in the sense that you can separate it from the process of investigation -, it remains that this very fact can be objectivized and explored in its connection with the other facts" Heisenberg.

The status of the formalisme of QM is - in the absence of first-hand objectivisation - to provide a theory of *second hand objectivisation*: a prediction tool, capable of generating the statistical distribution of facts obtained in the process of investigation, and which is *invariant to the process of investigation*. This tool is the state vector in the Hilbert space.

1. Epistemological motivation

We recognize the similarity between Human Sciences and Quantum Mechanic from an epistemological point of view: "the process of investigation is coextensive to the investigated object" Bitbol.

"*Analogously, - to classical physical measurement - the classical theory of preference assumes that each individual has a well-defined preference order and that different methods of elicitation produce the same ordering of options". But, "In these situations - of violation of procedural invariance - observed preferences are not simply read off from some master list; they are actually constructed in the elicitation process.*" in Kahneman and Tversky (2000)

Historically human sciences ($e.g.$, Hoffding) have been a model (in terms of epistemological situation) and inspired the development of QM (cf. Bohr).

2. From Knowledge to action

Any situation that requests a choice, an action, is an investigation a measuremnt of the agent's identity in the sens that behavior reveals individual preferences, attitudes, beliefs etc..

This is "the revealed preference argument": your choice is information about your preferences because we assume that you choose what you prefer.

⇒ As a consequence the epistemological argument applies to the analysis of behavior:

⇒Behavior is the outcome of a measurement of the agent's type(state) which is coextensive to the decision context.

3. Observational analogies

Many instances of "behavioral anomalies" can be viewed as phenomena of non-commutativity: order matters to the preferences that are being revealed.

Ex: cognitive dissonance: attitudes change with behavior: Festinger classical experience.

Other examples

- disjunction effect,
- inverse fallacy.
- preference reversal
- framing effects.

Parts of QM are clearly linked to the physical phenomena (Hamiltonian). But what parts of QM are purely the expression of the epistemological conditions:

- the probabilistic algorithm?
- the Hilbert space? or
- the orthocomplemented lattice structure of quantum logic?

Our view is that "quantum logic" as the most relevant part of the theory*. But it is little predictive. The "difficult issue" is the process of state transition under the impact of measurement.

We argue that the additional axiom needed to obtain the Hilbert space from quantum logic model is, though disputable, still acceptable in social sciences.

The basic framework: a theory of measurable system

1. The notions of measurement and of state

2. Basic structure on the state space

3. Discussion of relevance to social sciences

Measurements

A measurement is an interaction between a system and a measurement instrument. A measurement M results in an outcome $o \in O(M)$. We denote $\mathbf{M}$ the set of all measurements, $M \in \mathbf{M}$ is described by a mapping

$$\mu_M : \mathbf{P} \to \Delta(O(M))$$

The *number* $\mu_M(o|s)$ is the prob. of o when the state is $s \in \mathbf{P}$ (the set of pure state*).

First-Kindness

A measurement M satisfies the first-kindness property iff whenever it yields outcome o, then perfoming M again on the same system we obtain o with certainty. (First-kindness "fails": evolution*, noise, *M* is a combination of incompatible msnts).

Compatibility

M and N are compatible $\Leftrightarrow$ $M \circ N$ is first-kind.

If all M are compatible they can be combined into a single compound measurement that tells us everything about the (state of) the system $\Rightarrow$ we get the classical model.

A distinctive feature of non-classical measurable system is the existence of incompatible measurements i.e., measurements that cannot be performed simultaneously. Instead the order of performence matters to the outcome. This is intimately related to the "impact of measurement" on the "state"***.***

State

All info we have about a system is encapsulated in the state s. It is the basis for making predictions about future measurements: s defines a point $\mu_M(s) \in \Delta(O(M))$.

Let M be a measurement and $A \subset O(M)$. Denote

$$E_M(A) = \{s \in \mathbf{S},\ \mu_M(A|s) := \sum_{o \in A} \mu_M(o|s) = 1\}.$$

$E_M(A)$ is the set of states characterized by the fact that the result of M belongs to A for sure (eigenset of M). Such sets are called *properties*.

If $s;\ \mu_M(o|s) < 1,$ then after $M,$ the new state s' by fk $\mu_M(o|s') = 1$ so $s' \neq s$.

Pure and mixed states

The set of states $\mathbf{S} \subset \times_{M\in\mathbf{M}} \Delta(O(M))$. A pure state, state which is *not* a non-trivial mixture (convex comb.) of other states: $\mathbf{P} = \text{ext}(\mathbf{S})$.

In a classical system, pure states are dispersion-free, i.e., the outcome of any M is uniquelly determined. But generally pure states can also be "dispersed" ⇔ "intrinsic uncertainty" is related to

i. the existence of incompatible measurements and

ii. the impact of measurements on states.

- Indeed if all $s \in \mathbf{P}$ dispersion-free then no M ever impact on pure states ⇔ all M are compatible.
- While if $s \in \mathbf{P}$ is dispersed then by fk, s will be modified by an appropriate M and some M are incompatible.

A measurable system

A measurable system (ms) is a system equipped with a set of fk- measurements. A *model* of the ms is a collection of data:

1. A set of states $\mathbf{S}$;

2. An outcome mapping, $\mu_M \; : \; \mathbf{S} \to \Delta(O(M))$

3. A transition mapping, $\tau_{M,o} \; : \; \mathbf{S} \to \mathbf{S}$

Illustration: non-classical rational choice

A primitive measurement is a choice from a subset $A \subset X$; $O(A) = A$.

Assumptions: choice out of “small” subsets is

- well-defined i.e., first-kind

- rational: consecutive choices from “small” subsets satisfy Houthakker’s axiom (or IIA).

Definition: 1) Suppose the agent chooses from B an element a, and $a \in A \subset B$. If the consecutive measurement is A then the agent chooses a.

2) Suppose the agent chooses from A an element a. If the consecutive measurement is B then the outcome does not belong to $A \backslash \{a\}$.

We consider as "small" subsets of 3 or less.

A. All measurements commute

Model 0: From binary c-m we get an order $\prec$. From ternary c-m we see that it is transitive $\Rightarrow$ It is natural to identify **P** with the set of linear orders on X. We have the classical model!

B. We relaxe compatibility

Model 1: $X = \{a, b, c\}$ and 4 c-m $ab,\ ac,\ bc,\ abc$ all incompatible

$$\mathbf{P} = \{\underline{a}\,b,\ a\,\underline{b},\ \underline{a}\,c,\ a\,\underline{c},\ \underline{b}\,c,\ b\,\underline{c},\ \underline{a}\,bc,\ a\,\underline{b}\,c,\ ab\,\underline{c}\}.$$

To define the model of the ms we need the outcome and state transition mappings

If $s = a\,\underline{b}\,c$. $abc \rightarrow a\,\underline{b}\,c$, $ab \rightarrow a\,\underline{b}$, $bc \rightarrow \underline{b}\,c$

But what if we apply ac on s? $\rightarrow$ 1/2 $\underline{a}\,c$ or 1/2 $a\,\underline{c}$.

If $s = a\,\underline{b}$, $abc \rightarrow$ 1/3 $ab\,\underline{c}$ or 2/3 $a\,\underline{b}\,c$, $ac \rightarrow 2/3 a\,\underline{c}$ 1/3 $\underline{a}\,c$ etc...

Houthakker satisfied but no linear ordering

ex: $\underline{b}\,c$, $ac\left(\underline{b}\,c\right) = a\,\underline{c}$, $abc\left(a\,\underline{c}\right) = ab\,\underline{c}$

We need additionnal structure on the measurable system!

Basic structure on the state space

Orthogonality

If we split the set $O(M)$ in two parts, $O(+)$ and $O(-)$, we obtain a dichotomic measurement Q with two outcomes $+$ and $-$ we call it a question.

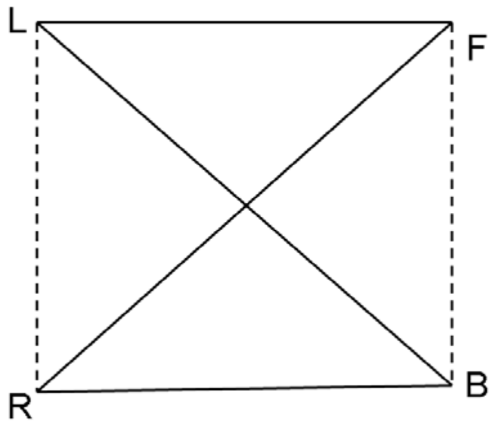
We say that two states s and t are orthogonal, $s \perp t$ if there exists a question Q : $(F, F^{\perp})$ such that $s(F) = 1$ and $t(F) = 0$.

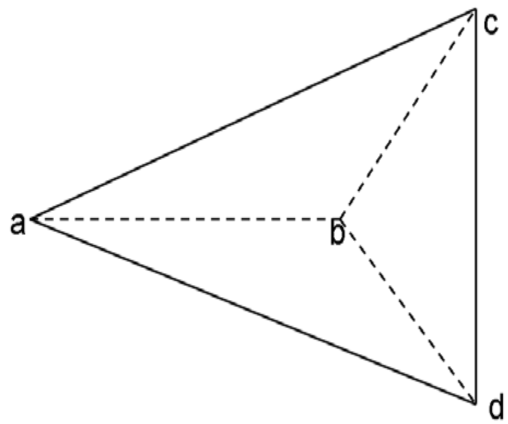
An orthogonality relation on a set X is a symmetric and irreflexive binary relation $\perp \subset X \times X$.

Definition *A set X equipped with an orthogonality relation $\perp$ is called an orthospace.*

The $\perp$ relationship induced by measurements on the state space provides the state space with the full structure of a logic (ortholattice).

A set F is said to be ortho-closed (also called a *flat*) if $F = F^{\perp\perp}$ where $F^{\perp} = \{x \in X, x \perp F\}.$ An orthospace X is *'ortho-separable'* when all the singleton sets are flats (orthoclosed).





The o-space in Fig 5 is not ortho-separable: $a^{\perp\perp} = \{a, c, d\} \neq a.$

Axiom *3: For any state s* $\in \mathbf{P}$, *the set* $\{s\}$ *is a property (ortho-closed subset).*

Substantive assumption: Any pure state can be 'prepared to'. Alternatively the set of pure state only contains elements that are testable. Violated in Model 1 $b\underline{c} \in \mathbf{P}$ but is not the eigenset of any M. the property $b\underline{c} = \{b\underline{c}, ab\underline{c}\}$

$\Longrightarrow$ Atomicity of the lattice of properties.

$\Longrightarrow$ The set of state $\mathbf{P}$ is ortho-separable. Ortho-separability is a *relaxation* of classical orthogonality, it allows for some *connectedness* between states.

$\Longrightarrow$ Let s and t be two pure states, due to axiom 3, we can speak about $s(t)$, i.e., the **probability for a transition** from s to $\{t\}$.

Axiom ***4*** *If two properties* P *and* Q *are comparable then there exists a measurement* $M \in \mathbf{M}$ *such that* $P = E_M(A)$ *and* $Q = E_M(B)$ *for some* $A, B \subset O(M)$.

It is violated in Model 1. $Q = c^{\perp}$ and $P = E_{ac}(\underline{a}\, c)$, $P \subset Q$ but $\nexists$ M with P and Q as its eigensets.

$\Longrightarrow$ The lattice of property is orthomodular: $F \leq G \Rightarrow G = F \vee \left(G \wedge F^{\perp}\right)$

$\Longrightarrow$ States are probability measures on the lattice of properties;

With some more "details" this where quantum logic "stops": an atomistic ortholattice of properties. The state space is divided into classes of connected elements (irreducible components). But we have little structure on the impact of measurement!

Impact of measurements

Assume $s \underset{\rightarrow}{Q} s',$ and we learned $s' \in F$ can we say anything more about the state s'?

Appealing to orthogonality we expect s' to be an ortho-projection of s on $F : s' \in F \wedge (s \vee F^{\perp})$.

Ideality **(least perturbation principle)**.

$Q(F, F^{\perp})$ is called ideal if for every state s, the new state $\tau_{M,o} \in F \wedge (s \vee F^{\perp})$. An ideal M conserves any property compatible with Q. (P compatible if $P \subset F$ or $P \subset F^{\perp}$).

$\Rightarrow$ If S belongs to an eigenset of M, applying M leaves s unaffected $\Leftrightarrow$ minimal perturbation.

But $F \wedge (s \vee F^{\perp})$ *may not be an atom!* creating a problem of predictability

We impose a last axiom

Axiom *6: For any pure state* $s \in \mathbf{P}$ *and any flat* F *the flat* $F \wedge (s \vee F^{\perp})$ *is an atom of the lattice* $\boldsymbol{F}(\mathbf{P}, \perp)$

Under the impact of measurement any pure state $s \in \mathbf{P}$ jumps into another pure state in $\mathbf{P}$.

$\Rightarrow$ We know precisely where a measurement takes the state.

$\Rightarrow$ Axiom 6 takes us with a leap toward the HSM of QM.

Discussion of the relevance of the basic structure to Social sciences

- An individual is a measureable system
- She is characterized by her type(state) that encapsulates all information about preferences, beliefs, attitude etc...
- A decision situation or a questionnaire is a device that measures her type;
- Actual behavior action taken in a game, choice made in a DS, response to a questionnaire are measurement outcomes.

The structural properties in social sciences

First kindness

In standard DT we assume repeatability, this is less demanding but may still be questionned.

Incompatible measurements

The distinctive feature of the non classical theory of measurement

1. Gives a precise sense to limitations of an individual ability to make comparisons on the universal set of items: all items cannot be compared simultaneously.
2. It links up with context dependency

$\Rightarrow$Captures bounded rationality consistently with 2 central themes of BE.

Ortho-separability and irreducibility

Ortho-separability: non-orthogonal i.e., connected pure states allowed. An irreducible system is fully connected. Under the impact of measurements any state can transit from any one state to any other.

⇒The framework allows the "construction of the type" in the process of elicitation. Because of non-orthogonality i.e., connectedness, the type of the agent changes under the impact of measurement.

A non-classical individual is structurally ‘plastic’.

Dynamics of measurements

The process of state transition in QM reflect two features:

1. Minimally perturbation principle (ideality)

2. No net loss of information (Axiom 6): A pure state transits into another pure state .

$\Rightarrow$ The impact of measurement is an orthogonal projection.

$\Rightarrow$ Behavioral types exhibit some stability: When asked to choose out of an initial state of hesitation, hesitation is only resolved so as to be able to produce an answer but not more. The remaining indeterminacy is left ‘untouched’.

This is far from innocuous, the act of choice could fully upset her previous state.

Yet, if we accept the minimal perturbation principle $\Rightarrow$ Type Indeterminacy Model

Type Indeterminacy

A Hilbert Space Model of Preferences and Choices

The notion of state

An agent is characterized by his *state* which encapsulates all info. about the agent's potential behaviors (preference system). It is represented by a vector $|\psi\rangle$ in a Hilbert space H over R. (M. Soler's theorem).

After having made a choice in a decision situation (DS), the state of the agent corresponds to the type (behavior) associated with the choice* he has made.

Ex: A DG* (50:50 (G) and 90:10 (E)) after choosing the type is either G or E.

‘Actualizing type G’ and ‘actualizing type E’ are two mutually exclusive events. In the Hilbert space model we propose this is captured in the *subspace structure* of the state space H.

$$H_G \oplus H_E = H, \; H_G \perp H_E$$

i.e to each possible type we associate a subspace which is pairwise orthogonal to the subspace associated with the other possible types. A decision situation (DS) is thus represented as a *resolution* of the state space H.

Prior to the decision, the agent’s preferences maybe indeterminate (he is hesitating) i.e. the state does not correspond to any eigentype .

The agent is then represented by a *superposition* (linear combination) e.g. of the two possible types of the DG:

$$|\psi\rangle = \lambda_1|G\rangle + \lambda_2|E\rangle, \quad \#$$

$\lambda_1, \lambda_2 \in \mathrm{R}, \quad \lambda_1^2 + \lambda_2^2 = 1. (*)$

Generally let $|\psi\rangle,\ |\varphi\rangle \in \mathrm{H}$ be two states, then any linear combination is a possible state for the individual. The *superposed* state: $[\lambda_1|\psi\rangle + \lambda_2|\varphi\rangle]$ does not generally correspond to a specific type however:

The state space is richer than the classical type space. This is the mathematical expression of indeterminacy.

The notion of operator and of measurement

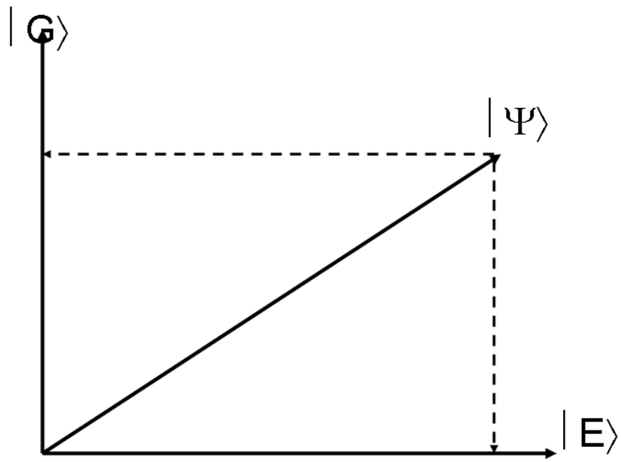
A decision situation, DS, is defined by the set of alternative choices available to the agent in a given situation. We here focus on simple decision situations i.e. non-strategic non repeated e.g.

Example *the choice between sure gain or gamble, tea /coffee, invest or not invest in x, DG, PD*

A DS can be thought of as an **experimental set-up** corresponding to a situation where the individual is invited to choose among the alternatives of the DS.

To each DS we associate an (Hermitian) operator called an *observable.**

The result of the experiment, can only be one of the eigenvalues of the observable (e.g 0 for E and 1 for G in the DG). The observable acts on the state vector which corresponds to the projection of the state onto one of the eigenspaces of the observable (corresponding to observed eigenvalue):



$|\Psi\rangle$ (DG) $\Rightarrow$ $|E\rangle$ with prob. λ_E

A single Decision Situation

More generally, when dealing with a single DS call it A, we can adopt its dimensionality. Its eigenvectors $|1\rangle, |2\rangle, \ldots, |n\rangle$ all correspond to different eigenvalues (only labels) $1, 2, .., $ and we can write

$$|\psi\rangle = \sum_{k=1}^{n} \lambda_k |k\rangle,$$

where $\lambda_i \in \mathbf{R}, \ \sum \lambda_i^2 = 1.$

Exposing the agent $|\psi\rangle$ to decision situation $A \Leftrightarrow$ 'measuring' the eigenvalue of $A \Leftrightarrow$ letting operator $A = \sum_{k=1}^{n} kP_k$ act on the state vector $|\psi\rangle$ as follows

$\Rightarrow$ the superposition $\sum_{k=1}^{n} \lambda_k |k\rangle$ collapses on one of its component say i. With probability equal to

$$\langle i|\psi \rangle^2 = \lambda_i^2$$

the state $|\psi\rangle$ collapses to $|i\rangle$ and the result of the measurement will be i.

For a single DS, the predictions of the HSM are the same as those of in the prob. model (square of the coefficients of the superposition instead of coefficients of the convex comb).

Two or more Decision Situations

A key issue whether the operators representing the DSs pairwise commute or not. *This is an empirical question!**

Two commuting DS

Example *A: (coffe or tea) and B: (invest or not in project x)*

When operators A *and* B commute there is an orthonormal basis of the relevant Hilbert space formed with *eigenvectors common to* A *and* B. This implies:

$$p_{AB}(i_A) = p_A(i_A)\forall i, \text{ and } p_{BA}(j_B) = p_B(j_B)\forall j$$

Again the HSM is equivalent with the probabilistic model.

Non-commuting Decision Situations

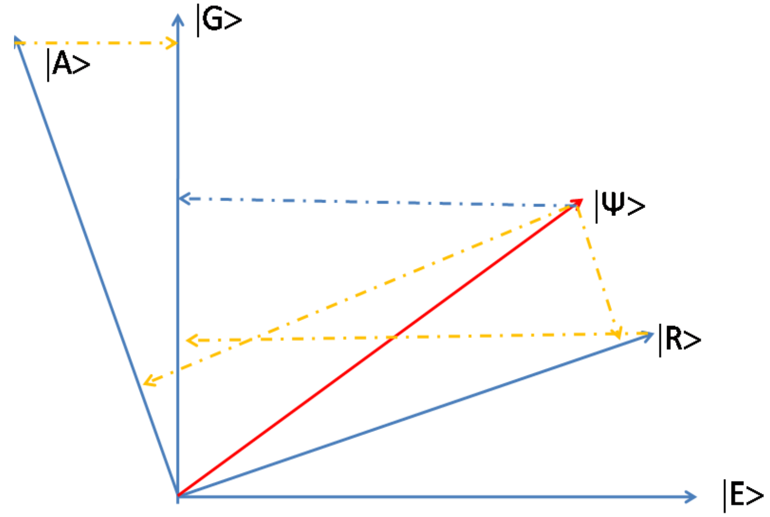
Let A and B have the same number n of possible choices. Their eigenvectors $\{|1_A\rangle, |2_A\rangle, ..|n_A\rangle\}$ and$\{|1_B\rangle, |2_B\rangle, ..|n_B\rangle\}$ form two different (orthonormal) basis of the same Hilbert space. Let $|\psi\rangle$ be the initial state of the player:

$$|\psi\rangle = \sum_{j=1}^{n} \lambda_i |i_A\rangle = \sum_{j=1}^{n} \gamma_j |j_B\rangle$$

The eigenvectors of B can be written in the basis made out of A's eigenvectors:

$$|j_B\rangle = \sum_{i=1}^{n} \mu_{ij} |i_A\rangle.$$

2 **non-commuting DS** DG = $\{G, E\}$, UG=$\{A(\text{accept}), R(\text{refuse})\}$



2 non-compatible measurements

$$|\Psi\rangle = \lambda_1|G\rangle + \lambda_2|E\rangle = \gamma_1|A\rangle + \gamma_2|R\rangle,\ |G\rangle = \delta_{1G}|A\rangle + \delta_{2G}|R\rangle,\ |E\rangle = \delta_{1E}|A\rangle + \delta_{2E}|R\rangle$$

One step measurement of UG

$$|\Psi\rangle = \gamma_1|A\rangle + \gamma_2|R\rangle \quad \Rightarrow prob_{UG}(A) = |\langle A|\Psi\rangle|^2 = \gamma_1^2$$

Or $\quad |\Psi\rangle = \lambda_1[\delta_{1G}|A\rangle + \delta_{2G}|R\rangle] + \lambda_2[\delta_{1E}|A\rangle + \delta_{2E}|R\rangle]$

$$\Rightarrow prob_{UG}(A) = \langle A|\Psi\rangle^2 = (\lambda_1\delta_{1G} + \lambda_2\delta_{1E})^2 (= \gamma_1^2)$$

Two steps measurement of UG:

First DG: $prob(G) = \langle G|\Psi\rangle^2 = \lambda_1^2,\ prob(E) = \langle E|\Psi\rangle^2 = \lambda_2^2$

Then UG: $prob(A|G) = \langle A|G\rangle^2 = \delta_{1G}^2\ , \quad prob(A|E) = \langle A|E\rangle^2 = \delta_{1E}^2$

$$prob_{UG,DG}(A) = \lambda_1^2\delta_{1G}^2 + \lambda_2^2\delta_{1E}^2 \neq (\lambda_1\delta_{1G} + \lambda_2\delta_{1E})^2$$

$$\Rightarrow prob_{UG}(A) \neq prob_{UG,DG}(A)$$

Returning to the general case:

$$|\psi\rangle = \sum_{j=1}^{n} \gamma_j |j_B\rangle = \sum_{j=1}^{n}\sum_{i=1}^{n} \gamma_j \mu_{ij} |i_A\rangle$$

If the player plays DS A in one step, he will play the choice i_A with the probability

$$p_A(i_A) = \left(\sum_{j=1}^{n} \gamma_j \mu_{ij} \right)^2$$

Playing DS B first, changes the way DS A is played:

$$p_{AB}(i_A) = \sum_{j=1}^{n} p_B(j_B)\, p(i_A|j_B) = \sum_{j=1}^{n} \gamma_j^2 \mu_{ij}^2 \qquad \#$$

The difference stems from the so called *interference terms:*

$$p_A(j_A) = \left(\sum_{j=1}^{n} \gamma_j \mu_{ij}\right)^2 = \sum_{j=1}^{n} \gamma_j^2 \mu_{ji}^2 + \underbrace{2 \sum_{j \neq j'} (\gamma_{j'} \mu_{ij})\left(\gamma_j \ \mu_{ij'}\right)}_{\text{Interference terms}}$$

$$p_A(j_A) = p_{AB}(i_A) - \text{interference terms}$$

Suggestive interpretation for interference effect :

Competing propensities to act coexist : the agent has not ‘made up his mind’ he hesitates. Those propensities interact i.e. they reinforce or neutralize each other in the process of determination when the agent is forced to choose.

The predictions of the TI-model are different from those of the probabilistic model: *indeterminacy reflects contextuality not (only) incomplete information.*

What structure does TI-model give?

1. Distinguishes beween 2 classes of DS, commuting and non-commuting.

2. Non-commuting A and B are linked:

$$|\psi\rangle = \begin{pmatrix}\lambda_1 & \lambda_2\end{pmatrix}\begin{pmatrix}|1_A\rangle \\ |2_A\rangle\end{pmatrix} = \begin{pmatrix}\lambda_1 & \lambda_2\end{pmatrix} S \begin{pmatrix}|1_B\rangle \\ |2_B\rangle\end{pmatrix}$$ where

$$S = \begin{pmatrix}\langle 1_B|1_A\rangle & \langle 2_B|1_A\rangle \\ \langle 1_B|2_A\rangle & \langle 2_B|2_A\rangle\end{pmatrix}.$$

The elements of S do have an interpretation: as (the square root of) the statistical correlations between DS-types. These *invariants* (independent of individual $|\psi\rangle$!) can be empirically estimated and used to calibrate predictive models of choice behavior.

Cognitive dissonance

$A = \{a_1, a_2\}$: decision about jobs:

a_1 : hazardous job (adventurous type),

a_2 : safe job (habit prone type).

$B = \{b_1, b_2\}$: use of safety equipement

b_1 : yes (risk avert type),

b_2 : No (risk loving type).

The observed CD phenomenon is that the probability that a person use safety equipement is lower if a decision type A is made before the decision to use or not safety equiment.

First scenario: The hazardous task is introduced in an existing context only B measured:

$$|\psi\rangle = \lambda_1|a_1\rangle + \lambda_2|a_2\rangle,\ \lambda_1^2 + \lambda_2^2 = 1.$$

and in the B basis

$$|\psi\rangle = \left[\lambda_1\langle b_1|a_1\rangle + \lambda_2\langle b_1|a_2\rangle \quad \right]|b_1\rangle + \\ + \left[\lambda_1\langle b_2|a_1\rangle + \lambda_2\langle b_2|a_2\rangle\right]|b_2\rangle$$

which gives

$$p_B(b_1) = \lambda_1^2\langle b_1|a_1(B)\rangle^2 + \lambda_2^2\langle b_1|a_2(B)\rangle^2 \\ + 2\lambda_1\lambda_2\langle b_1|a_2(B)\rangle\langle b_1|a_1(B)\rangle$$

Second scenario: first A then B^*.

$$p_{BA}(b_1) = p_A(a_1)p_B(b_1|a_1) + p_A(a_2)p_B(b_1|a_2)$$

$$p_{BA}(b_1) = \lambda_1^2\langle b_1|a_1(B)\rangle^2 + \lambda_2^2\langle b_1|a_2(B)\rangle^2$$

Whenever

$$p_B(b_1) > p_{BA}(b_1)$$

$$\Leftrightarrow 2\lambda_1\lambda_2\langle b_1|a_2(B)\rangle\langle b_1|a_1(B)\rangle > 0$$

we have a 'CD phenomenon'.

Numerical Example

Hyp: $|\psi\rangle = |b_1\rangle,\ \langle b_1|a_1\rangle = 0.5,\ \langle b_1|a_2\rangle = 0.866$

First scenario:

$$|\langle b_1|\psi\rangle|^2 = \left(\sqrt{0.75}\,\langle b_1|a_1\rangle + \sqrt{0.25}\,\langle b_1|a_2\rangle\right)^2$$

$$p_B(b_1) = 0.5625 + 0.0625 + 2 \bullet 0.375 = 1,$$

Second scenario:

$$p_{BA}(b_1) = 0.625 < p_B(b_1)$$

A contribution of the TI-model is to feature a dynamic process (resolution of indeterminacy) such that the propensity to use safety measure is actually reduced as a consequence of the act of choice.

FIRST CONCLUSIONS

- The TI-model formalizes the epistemological situation of human sciences after imposing axiom 6.
- The TI-model is a model where preferences are being ‘constructed’ not merely revealed.
- The TI-model provides with a general tool of prediction able to accommodate a variety of empirically observed phenomena. As such it has the potential of unifying different behavioral theories.

An intelligent agent understands that action/choice impacts on the state so she

1. may exploit to influence others in interactive situations $\Rightarrow$ individuals are endogenous to social interaction;

2. make choices to influence her own future preferences $\Rightarrow$ individual create/manage their own identity.

Games with Type Indeterminate Players

A game is an interactive situation where the agents' choice of action depends on what other do (or are expected to do).

From a formal point of view, introducing Type Indeterminacy in games essentially (but not only*) amounts to substituting the classical Harsanyi type space with another algebraic structure: A Hilbert space of self adjoint operators. (compare Q-games). A non-Bayesian updating rule consistent with the TI type space structure is formulated and an equilibrium concept Perfect TI-equilibrium.

⇒The TI-hypothesis extends the field of strategic interactions: actions impact not only on information and payoffs but also on the profile of types, i.e., on who the players are.

TI-games: general features

- We denote by *GS* an observable that measures the type of a player in a game. The interpretation of the outcome is that the chosen action is a *best reply* against the opponent's expected* action.
- In any specific TI-game *M* we must distinguish between the type which is identified with the player and the *eigentypes* (selves) which are identified with the payoff functions in game *M*.(simultaneous multiple selves)
- The reasoning leading to the determination of the best-reply is performed at the level of the *eigentypes* of the game.

A single interaction

Consider a 2X2 symmetric game, M:$\{L, R\}$, with the M-eigentypes:

θ_1 : prefers to go Left whatever he expects the opponent to do (Stubborn);
θ_2 : prefers to do as the other player (coordination C-type); θ_3 : prefers to do the contrary of the opponent("AC-type").

ex

θ_1/θ_2	L	R
L	10,12	10,3
R	0,3	0,12

θ_1/θ_3	L	R
L	10,0	10,10
R	0,10	0,0

.

Player 1:$|t_1\rangle = \lambda_1|\theta_1\rangle + \lambda_2|\theta_2\rangle + \lambda_3|\theta_3\rangle, \sum \lambda_i^2 = 1.$

If P1 plays with a $|t_2\rangle = |\theta_1\rangle$ with prob. $\lambda_1^2 + \lambda_2^2$ he plays *L* and (by L - v N) $|t_1\rangle \to |t_1'\rangle$

$$|t_1'\rangle = \frac{\lambda_1}{\sqrt{\lambda_1^2+\lambda_2^2}}|\theta_1\rangle + \frac{\lambda_2}{\sqrt{\lambda_1^2+\lambda_2^2}}|\theta_2\rangle$$

with prob.λ_3^2 he plays R and $|t_1\rangle \to |\theta_3\rangle$.

If $|t_2\rangle = |\theta_3\rangle$ who plays R, $|t_1\rangle \to |t_1''\rangle = \frac{\lambda_2}{\sqrt{\lambda_2^2+\lambda_3^2}}|\theta_1\rangle + \frac{\lambda_3}{\sqrt{\lambda_2^2+\lambda_3^2}}|\theta_3\rangle$

or $|t_1\rangle \to |\theta_2\rangle$

The resulting type depends on the opponent's type and corresponding expected play. Equivalent to a classical info interpretation of revised beliefs about player 1.

We now look for a TI-equilibrium for $\lambda_1 = \lambda_2 = \lambda_3 = \sqrt{.33}$ and $\gamma_1 \simeq 0$, $\gamma_2 \simeq \sqrt{0.4}$ and $\gamma_3 = \sqrt{.6}$.

A pure strategy TI-equilibrium of a two-player static game is a profile of strategies so each eigentype plays a best reply against the expected play of the opponent.

The following profile form a pure strategy TI-equilibrium as well as a PBE :

$$[\{(\theta_1^1, L), (\theta_2^1, R), (\theta_3^1, L)\}, \{(\theta_1^2, L), (\theta_2^2, L), (\theta_3^2, R)\}]$$

Prop 1: A pure strategy TI-eq of a max. info TI-game is equivalent to a PBE of the corresponding incomplete info game.

A simple multi-stage TI-game

The M is followed by "game" N, player 2 has the choice between Up and Down.

τ_1 : prefers Up ;

τ_2 : prefers Down.

Simple decision types, preferences of the types of P1 independent of play at stage 1.

$$|\theta_1\rangle = \alpha_1|\tau_1\rangle + \alpha_2|\tau_2\rangle; \quad |\theta_2\rangle = \beta_1|\tau_1\rangle + \beta_2|\tau_2\rangle;$$

$$|\theta_3\rangle = \delta_1|\tau_1\rangle + \delta_2|\tau_2\rangle$$

We assume $\delta_1 = 0$, and **P2's (end)payoff depends critically on P1's stage 2 decision**: for any path of stage 1, his payoff of Up is between 100 and 150 and for D is 0 for all paths of stage 1.

We now add stage 2 and we want to compare the PBE and the PTIE for the whole game.

In the corresponding classical incomplete information model we have type space $\{\theta_1\tau_1, \theta_2\tau_1, \theta_3\tau_1, \theta_1\tau_2, \theta_2\tau_2, \theta_3\tau_2\}$. The initial beliefs are given by the square of the coefficients of superposition.

$$prob(U \| t_1 \rangle |) = \lambda_1^2 \alpha_1^2 + \lambda_2^2 \beta_2^2.$$

It does **not** depend on the history of play at stage 1 only on the type of player 1. $\Rightarrow$ the eq. of stage 1 alone is part for the PBE of the whole game.

What about the TI-model?

We show that P2 can do better than in the PBE. He can increase the prob for the play of Up (by P1). If θ_3^2 chooses L, the θ_2^1 best-reponds by choosing L i.e., pooling with θ_1^1. The resulting type is

$$|t_1'\rangle = \frac{\lambda_1}{\sqrt{\lambda_1^2+\lambda_2^2}}|\theta_1\rangle + \frac{\lambda_2}{\sqrt{\lambda_1^2+\lambda_2^2}}|\theta_2\rangle$$

with prob $\lambda_1^2+\lambda_2^2$ and $\left|\theta_3\right\rangle$ with probλ_3^2.

$$prob(U;|t_1\rangle) = prob(U;|t_1'\rangle) = \lambda_1^2\alpha_1^2+\lambda_2^2\beta_1^2 + 2\lambda_1\alpha_1\lambda_2\beta_1 > \lambda_1^2\alpha_1^2+\lambda_2^2\beta$$

Intuition

In the classical Bayes-Harsanyi version of the game, player 2 has no means of influencing player 1's play at stage 2. This play depends exclusively on the type of player 1, which is known to player 1 from the beginning of the game.

In the TI-version of this game,

1. the expected move of player 2 at stage 1 induces a measurement of player 1's indeterminate type.

2. The measurement changes the type of player 1 by inducing some patterns of separation (or pooling) between eigentypes.

3. In our example when indeterminacy is preserved (by pooling) between θ_1^1, θ_2^1, this gives rise to interference effects at stage 2 which increase the probability for Up.

Key Features of TI-game: strategic manipulation

- Type indeterminacy implies that *a move has impact* not only on information and payoffs but also *on the the type of the opponent* who best-replies to the move.
- The nature of the impact is to *induce or not induce separation* between possible eigentypes.
- If the eigentypes pool, interference effects affect subsequent play while if they separate the interference effects are destroyed leading to another probability distribution over subsequent play.
- Players may have a strategic interest in inducing (by their own move) separation or preserving superposition. This leads to *strategic "manipulation"* as part of a Perfect TI-equilibrium.

Do TI-games bring any news for Economist?

The hidden variable argument

1. The first answer is NO: There exists a series of impossibility results: "the algebraic structure of QM *cannot* be embedded into a commutative algebra of real valued functions on a phase space of HV" (no Physics only Maths).

(von Neuman, Joch and Piron and Kochen Specher)

2. The second answer is YES: HV theories can often be constructed if you give up the classical paradigm: properties do not belong to a system but to a system *in a context.* (contextual HV).

What about GT and Economics?

- GT at it most general level is consistent with contextual types (2). But in applications the classical paradigm is maintained: agents(types) are defined separately from the context in which they interact.

⇒ In economics result (1) apply: the predictions of TI-games cannot generally be obtained by extending the model i.e., it is truly novel.

Examples of possible applications

- Impact of pre-play on the selection principle in multiple equilibria situations (risk dominance, payoff dominance, avoidance of loss etc...) Experiments show that a pre-play auction for the right to play a coordination game tends to lead to loss avoidance.
- Selection of reference point in bargaining. (Winning a unrelated contest before playing a ultimatum game affects the offer and rejection threshold)
- The sunk cost fallacy: the act of buying a subscription to the theater impacts on your preferences so you go more often.
- Path dependence: a move can has far reaching consequences for subsequent play e.g., when it radically changes the type.

Concluding remarks

- The objective has been to establish the approach of QM is the Social Sciences on a strong epistemological ground and with explicit link to measurements.
- The presentation focused on state transition due to measurement but time dynamics i.e., evolution has also been considered within this framework in particular in works relevant to psychology.
- Some view quantum probability as the core of QM to be exported and use it *without explicit reference to measurements* or other interactions to obtain often observed phenomena of super and subadditivity in jugment and choices.
- Still others have chosen to use standard quantum mechanics with formula directly borrowed from physics to fit data.

THANK YOU!