A quantum-like model of *Escherichia coli*'s metabolism based on adaptive dynamics

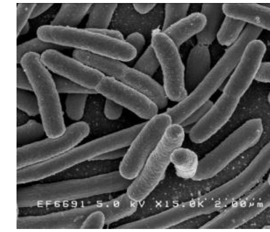
Quantum Interaction 2012, Paris, June 27-29

Masanari Asano, Irina Basieva, Andrei Khrennikov,

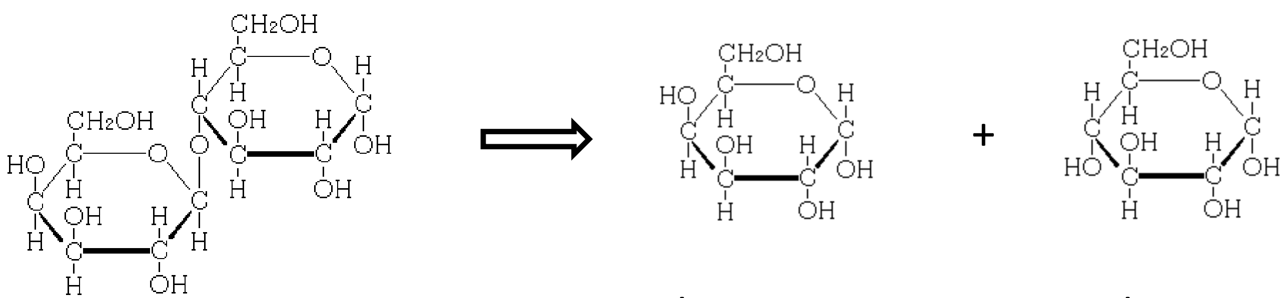
Masanori Ohya, †Yoshiharu TANAKA and Ichiro Yamato

† Dept. of Information Sciences, Tokyo Univ. of Science, Japan

Introduction



- In biology, the mechanism of glucose/lactose metabolism has been studied well with a bacterium, ***Escherichia coli (E.coli).***
- In metabolizing **lactose**, *E. coli* must produce the enzyme of **β-galactosidase**.



lactose glucose galactose

- *E.coli*'s production of β-galactosidase is adaptive to the context of the surroundings: Concentrations of glucose/lactose.

Ohya's talk: Adaptive dynamics

Glucose effect

It is well-known that *E. coli* metabolizes glucose in preference.
Such *E. coli*'s preference is seen in the following simple experiment.

<preparation> *E.coli* is incubated in different concentrations for several hours.

(i) *E.coli* + Lac 0.4%　　(ii) *E.coli* + Glu 0.4%　　(iii) *E.coli* + Lac 0.4% + Glu 0.4%

The activity of β-gal can be measured with some methods(e.g. β-gal assay).

<measurement>

	(i)	(ii)	(iii)
The degree of β-gal activity (Miller units)	**2132** / 2200	**56** / 2200	**64** / 2200

If glucose exists (even when its concentration is very low),
then E.coli ***does not produce*** *β-galactosidase.*

Violation of total probability law in glucose effect

<Event system1>

$L:$ E.coli detects lactose molecule

$G:$ E.coli detects glucose molecule

<Event system2>

$+:$ β-galactosidase active

$-:$ β-galactosidase not active

<Probabilities obtained from experimental setting and results>

Violation of TPL

$$P(+) \neq P(+|L)P(L) + P(+|G)P(G)$$

$$\frac{64}{2200} \neq \frac{2132}{2200} \cdot \frac{1}{2} + \frac{56}{2200} \cdot \frac{1}{2}$$

Test tube (iii): Lac 0.4% + Glu 0.4%

Test tube (i): Lac 0.4%

Test tube (ii): Glu 0.4%

[1] I.Basieva, A.Khrennikov, M.Ohya and I.Yamato, Syst. Synth. Biol., Springer, 1-10 (2011).

Experimental data for different strains and preculturing conditions

data	A	B	C	D	E
strain	W3110	W3110	ML30	ML308	ML308-2
***lacI** (repressor protein)*	+	+	+	-	-
***lacY** (lac. transporting sys.)*	+	+	+	+	-
Preculturing medium	LB	Gly	LB	LB	LB
(i) 0.4% lac	**2132**	**957**	**1763**	**2563**	**6140**
(ii) 0.4% glu	**56**	**5**	**14**	**1592**	**3062**
(iii) 0.4% glu+ 0.4% lac	**64**	**421**	**78**	**2050**	**862**

- W3110 and ML30 are wild type, and others are mutants.
- A, B, C and E show the violation of the total probability law.

New mathematical description of the glucose effect of *E.coli*

Now suppose that there are two general event systems

$$A = \{a_k \in \mathbb{R}, E_k \in \mathcal{A}\} \text{ and } B = \{b_j \in \mathbb{R}, F_j \in \mathcal{B}\}$$

where we do not assume E_j, F_k are projections but they satisfy the conditions $\sum_j E_j = I, \sum_k F_k = I$ as POVM (positive operator valued measure) corresponding to the partition of a probability space in classical system. Then the "joint-like" probability obtainning a_k and b_j might be given by

$$P(a_k, b_j) = \mathrm{tr} F_k \boxdot E_j \mathcal{E}^*_{\sigma Q} \rho,$$

where $\boxdot$ is a certain operation (relation) between A and B, more generally one can take a certain operator function $f(F_k, E_j)$ instead of $F_k \boxdot E_j$. (*e.g.* tensor product $\otimes$ in simple case)

$\mathcal{H} = \mathcal{K} = \mathbb{C}^2$

$\mathcal{A}, \mathcal{B}$: sets of all observables on Hilbert spaces $\mathcal{H}, \mathcal{K}$, resp.

$\mathcal{S}(\mathcal{H}), \mathcal{S}(\mathcal{K})$: sets of all states on Hilbert spaces $\mathcal{H}, \mathcal{K}$, resp.

General event system1

$\{a_k \in \mathbb{R}, E_k \in \mathcal{A}\}$ $\quad E_k = |e_k\rangle\langle e_k|$ $\quad \{|e_k\rangle\}$: CONS of $\mathcal{H}$

General event system2

$\{b_k \in \mathbb{R}, F_k \in \mathcal{B}\}$ $\quad F_k = |f_k\rangle\langle f_k|$ $\quad \{|f_k\rangle\}$: CONS of $\mathcal{K}$

<Event system1>

L: E.coli detects lactose molecule $\leftrightarrow E_1$

G: E.coli detects glucose molecule $\leftrightarrow E_2$

<Event system2>

$+$: β-galactosidase active $\leftrightarrow F_1$

$-$: β-galactosidase not active $\leftrightarrow F_2$

New mathematical law computing the probability in adaptive dynamics for glucose effect of *E.coli*

Initial state $\rho = |x_0\rangle\langle x_0|$ $\quad |x_0\rangle = \frac{1}{\sqrt{2}}|e_1\rangle + \frac{1}{\sqrt{2}}|e_2\rangle$

Lifting $\mathcal{E}^*_{D,Q} : \mathcal{S}(\mathcal{H}) \to \mathcal{S}(\mathcal{K} \otimes \mathcal{H})$

$$\mathcal{E}^*_{D,Q}\rho \equiv \rho_{D,Q} \otimes \rho_D$$

<u>Adaptive change</u>

Detection state $\rho_D \equiv \frac{D\rho D^*}{\mathrm{tr}(|D|^2 \rho)} \in \mathcal{S}(\mathcal{H})$

Activation state $\rho_{D,Q} \equiv \frac{Q\rho_D Q^*}{\mathrm{tr}(|Q|^2 \rho_D)} \in \mathcal{S}(\mathcal{K})$

Detection operator $D = \begin{pmatrix} \alpha & 0 \\ 0 & \beta \end{pmatrix}$

Activation operator $Q = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$$P(L) = |\alpha|^2, P(G) = |\beta|^2$$

[2] M. Asano, I. Basieva, A. Khrennikov, M. Ohya, Y. Tanaka and I. Yamato, Syst. Synth. Biol., DOI:10.1007/s11693-012-9091-1 (2012)

Formula computing Joint probabilities

$$P_{E_1}(+,L) = \mathrm{tr}\left[\left(F_1 \otimes E_1\right)\mathcal{E}^{*}_{E_1,Q}\rho\right],$$
$$P_{E_2}(+,G) = \mathrm{tr}\left[\left(F_1 \otimes E_2\right)\mathcal{E}^{*}_{E_2,Q}\rho\right],$$
$$P_{E_1}(-,L) = \mathrm{tr}\left[\left(F_2 \otimes E_1\right)\mathcal{E}^{*}_{E_1,Q}\rho\right],$$
$$P_{E_2}(-,G) = \mathrm{tr}\left[\left(F_2 \otimes E_2\right)\mathcal{E}^{*}_{E_2,Q}\rho\right],$$

$\longleftrightarrow$

$$P_D\left(+\,|\,L\right) = \mathrm{tr}\left(F_1 \otimes I\right)\mathcal{E}^{*}_{D,Q}(E_1)$$
$$P_D\left(+\,|\,G\right) = \mathrm{tr}\left(F_1 \otimes I\right)\mathcal{E}^{*}_{D,Q}(E_2)$$
$$P_D\left(-\,|\,L\right) = \mathrm{tr}\left(F_2 \otimes I\right)\mathcal{E}^{*}_{D,Q}(E_1)$$
$$P_D\left(-\,|\,G\right) = \mathrm{tr}\left(F_2 \otimes I\right)\mathcal{E}^{*}_{D,Q}(E_2)$$

$$P_D\left(+\right) \equiv \mathrm{tr}\left[\left(F_1 \otimes I\right)\mathcal{E}^{*}_{D,Q}\rho\right], \quad P_D\left(-\right) \equiv \mathrm{tr}\left[\left(F_2 \otimes I\right)\mathcal{E}^{*}_{D,Q}\rho\right]$$

With these probabilities, the total probability law is violated.

$$P_D(+) \neq P_D(+\,|\,L)P(L) + P_D(+\,|\,G)P(G)$$
$$P_D\left(+\right) \neq P_{E_1}(+,L) + P_{E_2}(+,G)$$

$$P_{E_1}(+,L)=P_D(+\,|\,L)=\frac{|a|^2}{|a|^2+|c|^2},\quad P_{E_2}(+,G)=P_D(+\,|\,G)=\frac{|b|^2}{|b|^2+|d|^2},$$

$$P_{E_1}(-,L)=P_D(-\,|\,L)=\frac{|c|^2}{|a|^2+|c|^2},\quad P_{E_2}(-,G)=P_D(-\,|\,G)=\frac{|d|^2}{|b|^2+|d|^2},$$

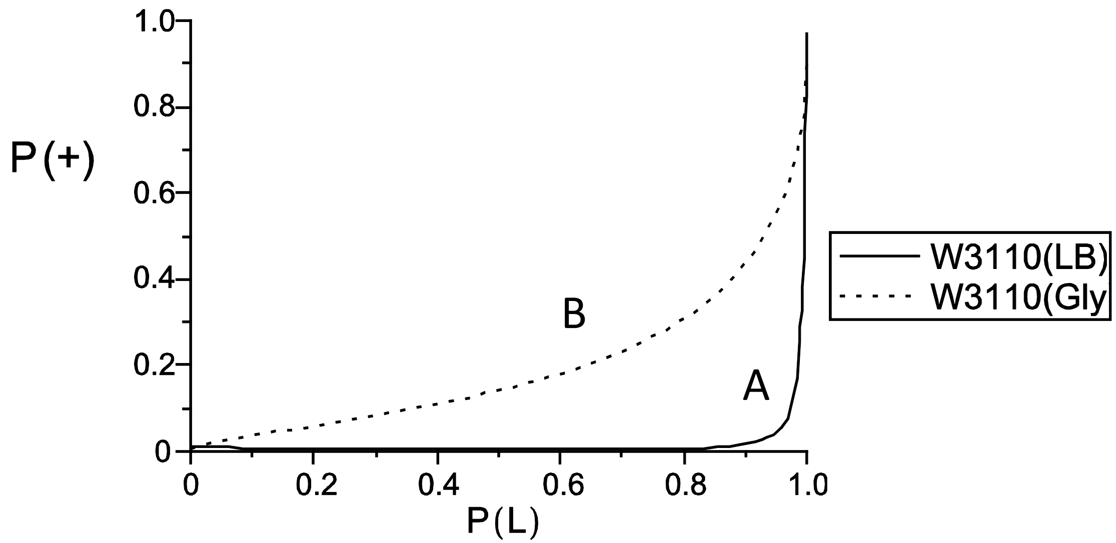
From these results, we may give the following forms for a, b, c and d.

$$a=\sqrt{P(+\,|\,L)}\mathrm{e}^{i\theta_{+L}}k_L,\ c=\sqrt{P(-\,|\,L)}\mathrm{e}^{i\theta_{-L}}k_L,$$
$$b=\sqrt{P(+\,|\,G)}\mathrm{e}^{i\theta_{+G}}k_G,\ d=\sqrt{P(-\,|\,G)}\mathrm{e}^{i\theta_{-G}}k_G.$$

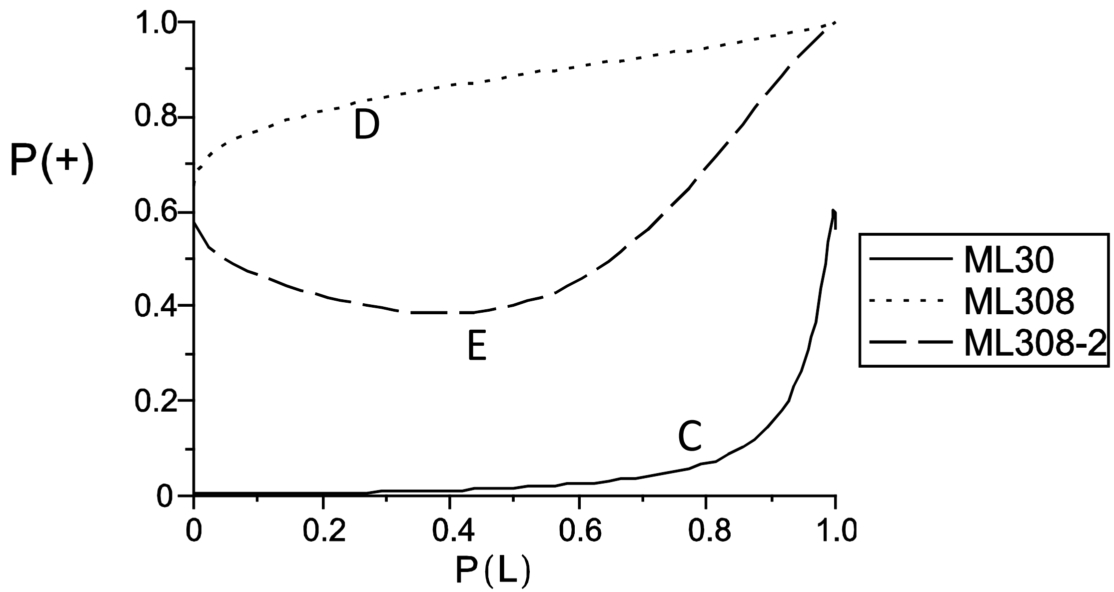
Here k_L and k_G are real number. Then the operator Q is decomposed to

$$Q=\begin{pmatrix}\sqrt{P(+\,|\,L)} & \sqrt{P(+\,|\,G)}\\ \sqrt{P(-\,|\,L)} & \sqrt{P(-\,|\,G)}\end{pmatrix}\begin{pmatrix}e^{i\theta_L}k_L & 0\\ 0 & e^{i\theta_G}k_G\end{pmatrix}$$

	A	B	C	D	E
$\sqrt{\frac{k_L}{k_G}}$	0.066	0.406	0.190	0.838	0.697
$\cos\theta$	–0.842	1.000	–0.461	0.251	–0.733



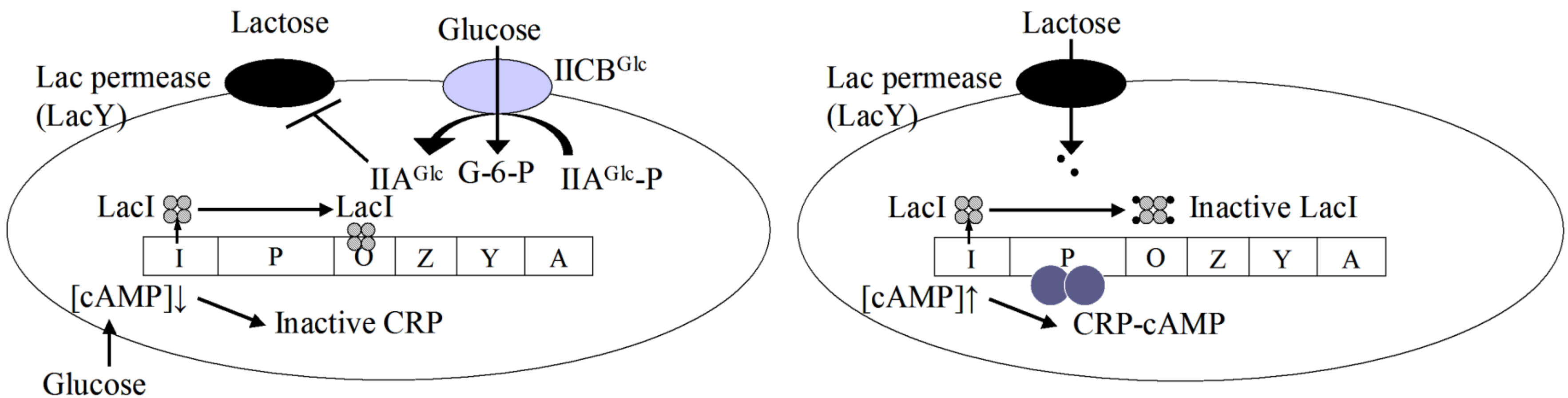
Effects of Preculturing condition



Effect of lacI and lacY genes

lac operon theory

- *lac* operon is a set of genes required for the transport and metabolism of lactose in *E.coli*.



(left) There is both glucose and lactose in the medium (right) There is only lactose.

Z ･･･ a gene of β-galactosidase

I ･･･ a gene of **“repressor” proteins (LacI)**

Y ･･･ a gene of **Lac permease(LacY)**

Conclusion

- We presented a new mathematical formula to compute the joint probability and conditional probability in contextual dependent adaptive systems by using the concepts of the adaptive dynamics.
- We realized the above fact in E.coli’s glucose effect.

Reference

[1] I.Basieva, A.Khrennikov, M.Ohya and I.Yamato, “Quantum-like interference effect in gene expression: glucose-lactose destructive interference”, Syst. Synth. Biol., Springer, 1-10 (2011).

[2] M. Asano, I. Basieva, A. Khrennikov, M. Ohya, Y. Tanaka and I. Yamato, “Quantum-like model for the adaptive dynamics of the genetic regulation of E. coli's metabolism of glucose/lactose”, Syst. Synth. Biol., DOI:10.1007/s11693-012-9091-1 (2012)

[3] M.Asano, I.Basieva, A.Khrennikov, M.Ohya and I.Yamato. “A general quantum information model for the contextual dependent systems breaking the classical probability law”,arXiv:1105.4769.

[4] M.Ohya and I.Volovich, “Mathematical Foundations of Quantum Information and Computation and Its Applications to Nano-and Bio-systems”, Springer-Verlag, (2011).

[5] L.Accardi, M.Ohya, “Compound Channels, Transition Expectations, and Liftings”, Appl. Math. Optim., 39, 33-59 (1999)