Hierarchical Bayesian Estimation of Quantum Decision Model Parameters

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June 27, 2012

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    - order effects in inference (Trueblood & Busemeyer, Cog Sci, 2011)
    - conjunction and disjunction fallacies (Busemeyer, et al., Psych Rev, 2011)

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- Bayesian model comparison provides a coherent method for comparing models with respect to both accuracy and parsimony
- This paper uses hierarchical Bayesian parameter estimation to investigate the parameter values of quantum and traditional decision models for a challenging set of data

Outline of paper

1. Describe phenomena and data

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4. Hierarchical Bayesian estimation of quantum parameters

Topic 1:

Describe phenomena and data

Savage's Sure Thing Principle

Suppose

- when S is the state of the world, you prefer action A over B.

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- when S is the state of the world, you prefer action A over B.
- when $\bar{S}$ is the state of the world, then you also prefer action A over B
- Therefore you should prefer A over B even when S is unknown

Test of the Sure Thing Principle

Tversky and Shafir (1992, Psych Sci)

- Person is faced with a gamble 'win $200 or lose $100'

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- Violates Sure thing principle and law of total probability
  - Prob play second when first is unknown = (prob win first $\times$ prob play given win) $+$ (prob lose first $\times$ prob play given loss) $>$ prob play given loss

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  - After an actual win, do you now choose to gamble on stage two?

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- Final decision
  - After an actual win, do you now choose to gamble on stage two?
  - After an actual loss, do you now choose to gamble on stage two?
- Randomly chose either the plan or the final to determine monetary payment

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- 100 participants

Barkan and Busemeyer (2003) Results

Gamble		Win First Play		Gamble		Lose First Play	
Win	Loss	Plan	Final	Win	Loss	Plan	Final
200	220	0.46	0.34	80	100	0.36	0.44
180	200	0.45	0.35	100	120	0.47	0.63
200	200	0.59	0.51	100	100	0.63	0.64
120	100	0.70	0.62	200	180	0.57	0.69
140	100	0.62	0.54	160	140	0.68	0.69
200	140	0.63	0.53	200	160	0.67	0.72
200	120	0.74	0.68	160	100	0.65	0.73
200	100	0.79	0.70	180	100	0.68	0.80
				200	100	.85	.82

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  - changed from planning to play again to finally not playing again
- After a loss, players become more risk seeking
  - changed from not planning to play again to finally playing again

Topic 2:

Describe models

Two competing models

- Quantum model

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- Markov model
  - Reduction of the quantum model when one key parameter is set to zero

Quantum model

- Four events correspond to four outcomes from two stage gamble: $\{|WT\rangle, |WR\rangle, |LT\rangle, |LR\rangle\}$

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  - W = win first gamble, L = lose first gamble
  - T = take second gamble, R = reject second gamble
- State of the decision maker is a superposition over these four orthonormal basis states:

$$|\psi\rangle = \psi_{WT} \cdot |WT\rangle + \psi_{WR} \cdot |WR\rangle + \psi_{LT} \cdot |LT\rangle + \psi_{LR} \cdot |LR\rangle$$

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- From first gamble to second gamble:

$$\psi_F = U \cdot \psi_I$$

$$U = \exp\left(-i \cdot \frac{\pi}{2} \cdot (H_1 + H_2)\right)$$

Quantum model

- Hamiltonian $= H_1 + H2$:

$$H_1 = \begin{bmatrix} \frac{h_W}{\sqrt{1+h_W^2}} & \frac{1}{\sqrt{1+h_W^2}} & 0 & 0 \\ \frac{1}{\sqrt{1+h_W^2}} & \frac{-h_W}{\sqrt{1+h_W^2}} & 0 & 0 \\ 0 & 0 & \frac{h_L}{\sqrt{1+h_L^2}} & \frac{1}{\sqrt{1+h_L^2}} \\ 0 & 0 & \frac{1}{\sqrt{1+h_L^2}} & \frac{-h_L}{\sqrt{1+h_L^2}} \end{bmatrix}$$

$$H_2 = \frac{-\gamma}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

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- h_W, h_L = utilities for taking the gamble or not; γ = free parameter allowing for changes in beliefs

Utility parameters

- utilities for taking the gamble after a win

$$h_W = \frac{2}{1 + e^{-D_W}} - 1$$

$$D_W = u(G|Win) - x_W^a$$

if $(x_W - x_L) > 0$:

$$u(G|Win) = (.50) \cdot (x_W + x_W)^a + (.50) \cdot (x_W - x_L)^a$$

if $(x_W - x_L) < 0$:

$$u(G|Win) = (.50) \cdot (x_W + x_W)^a - (.50) \cdot b \cdot |(x_W - x_L)|^a$$

Utility parameters

- utilities for taking the gamble after a loss

$$h_L = \frac{2}{1 + e^{-D_L}} - 1$$

$$D_L = u(G|Loss) - (-b \cdot x_L^a)$$

if $(x_W - x_L) > 0$:

$$u(G|Loss) = (.50) \cdot (x_W - x_L)^a - (.50) \cdot b \cdot (x_L + x_L)^a$$

if $(x_W - x_L) < 0$:

$$u(G|Loss) = -(.50) \cdot b \cdot |(x_W - x_L)|^a - (.50) \cdot b \cdot (x_L + x_L)^a$$

Quantum model

- Projection matrix for taking the gamble:

$$M = \begin{bmatrix} T & 0 \\ 0 & T \end{bmatrix}, \; T = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

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- ψ_I = first gamble unknown, ψ_W = win first gamble, ψ_L = lose first gamble
- The probability of planning to take the second stage gamble

$$p(T|Plan) = ||M \cdot U \cdot \psi_I||^2$$

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- The probability of planning to take the second stage gamble

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- The probability of taking the second stage game following a win

$$p(T|Win) = ||M \cdot U \cdot \psi_W||^2$$

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- The probability of taking the second stage game following a win

$$p(T|Win) = ||M \cdot U \cdot \psi_W||^2$$

- The probability of taking the second stage game following a loss

$$p(T|Loss) = ||M \cdot U \cdot \psi_L||^2$$

Quantum model

- If $\gamma \neq 0$ then the quantum model produces interference that accounts for dynamic inconsistency effects:

$$
\begin{aligned}
||M \cdot U \cdot \psi_I||^2 &= \frac{1}{2} \cdot ||M \cdot U \cdot (\psi_W + \psi_L)||^2 \\
&= \frac{1}{2} \cdot ||M \cdot U \cdot \psi_W + M \cdot U \cdot \psi_L||^2 \\
&= \frac{1}{2} \cdot ||M \cdot U \cdot \psi_W||^2 + \frac{1}{2} \cdot ||M \cdot U \cdot \psi_L||^2 \\
&\quad + \frac{1}{2} \cdot (\psi_W^\dagger \cdot U \cdot M) \cdot (M \cdot U \cdot \psi_L) \\
&\quad + \frac{1}{2} \cdot (\psi_L^\dagger \cdot U \cdot M) \cdot (M \cdot U \cdot \psi_W)
\end{aligned}
$$

Markov model

- The Markov model is a special case of the quantum model when $\gamma = 0$

Markov model

- The Markov model is a special case of the quantum model when $\gamma = 0$
- In this case ($\gamma = 0$) there are no interference effects

Comparing fits to mean data

- Fit to 33 means

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  - $R^2 = .82, SSE = .10 \ (a = .71, b = 2.5, \gamma = -4.4)$

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- Markov ($\gamma = 0$)

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  - Three parameters: a and b used to determine the utilities; γ for changing beliefs to align with actions.
  - $R^2 = .82, SSE = .10$ $(a = .71, b = 2.5, \gamma = -4.4)$
- Markov $(\gamma = 0)$
  - $R^2 = .78, SSE = .12$ $(a = .86, b = 2.3)$

Topic 3:

Log likelihood analysis

Log likelihood analysis of individual data

- Model for a single trial with choice pair (plan, final)

$$
\begin{aligned}
r &= \text{probability recall previous choice} \\
p_{TT} &= \Pr[T|plan] \cdot r + \Pr[T|plan] \cdot (1-r) \cdot \Pr[T|final] \\
p_{TR} &= \Pr[T|plan] \cdot (1-r) \cdot \Pr[R|final] \\
p_{RT} &= \Pr[R|plan] \cdot (1-r) \cdot \Pr[T|final] \\
p_{RR} &= \Pr[R|plan] \cdot r + \Pr[R|plan] \cdot (1-r) \cdot \Pr[R|final] \\
D_{jk}(t) &= 1 \textit{ if } (j,k) \text{ occurs on trial } t\text{, otherwise } D_{jk}(t) = 0 \\
\ln L(t) &= \sum D_{jk}(t) \cdot \ln(p_{jk})
\end{aligned}
$$

- log likelihood for individual i on all 33 pairs of trials

$$
\ln L(D_i) = \sum_{i=1}^{33} \ln L(t)
$$

Grid analysis of parameters

- Four parameters $\theta = (r, a, b, \gamma)$
- Used a grid of 21 points per parameter
  - 21^4combinations
- *Memory* r $\in$ [.00, .05, ..., .45, .500, .55, ..., .95, 1.00]
- Risk Aversion: a $\in$ [.400, .45, ..., .85, .90, .95, ..., 1.35, 1.40]
- *Loss* aversion: b $\in$ [.50, .60, ..., 1.40, 1.50, 1.60, ...2.40, 2.50]
- *Choice* parameter: $\gamma \in [-5.00, -4.5, ..., -.5, 0.0, .5, ..., 4.5, 5.00]$

Topic 4:

- Hierarchical Bayesian estimation of quantum parameters

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- Used to evaluate whether or not $H_0 : \gamma = 0$ for the quantum model

Hierarchical parameters

- θ_i := vector of four model parameters $\theta_i = (\theta_{i1}, \theta_{i2}, \theta_{i3}, \theta_{i4})$ for person i
- θ := vector for all participants, 400 parameters
- π := vector of four hierarchical parameters, $\pi = (\pi_1, \pi_2, \pi_3, \pi_4)$
- $L(D_i|\theta_i)$:= likelihood of data given model parms for person i
- $q(\theta_i|\pi)$:= prior probability of parameters for person i dependent on hierarchical parm's
- $r(\pi)$:= prior probability over hierarchical parameters

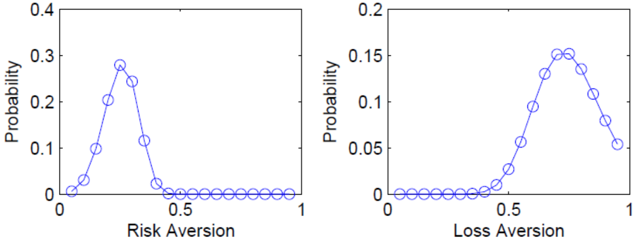
Specific Hierarchical Prior Assumptions

$$
\begin{aligned}
r(\pi) &= \prod_{j=1}^{4} r(\pi_j) \\
r(\pi_j) &= \mathit{uniform}\left[.05, .10, \ldots .50, .., .90, .95\right] \\
q(\theta_i|\pi) &= \prod_{j} q(\theta_{ij}|\pi_j) \\
q(\theta_{ij}|\pi_j) &= \mathit{bin}(\pi_j, 21)
\end{aligned}
$$

Distributions

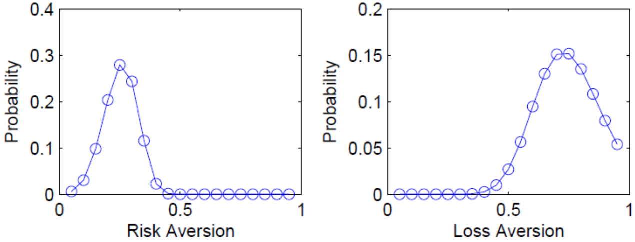
$$
\begin{aligned}
P(\pi, \theta, D) &= r(\pi) \cdot \prod_{i=1}^{N} q(\theta_i|\pi) \cdot L(D_i|\theta_i) \\
P(\pi, D) &= \sum_{\theta} P(\pi, \theta, D) \\
P(D) &= \sum_{\pi} P(\pi, D) \\
P(\pi|D) &= \frac{P(\pi, D)}{P(D)}
\end{aligned}
$$

HBA estimates of group level parameters



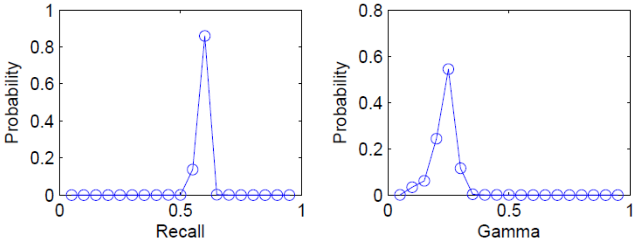
- The risk aversion hierarchical parameter distribution is located below .50 implying the mean of the risk aversion parameter equals .65, indicating somewhat strong risk aversion

HBA estimates of group level parameters



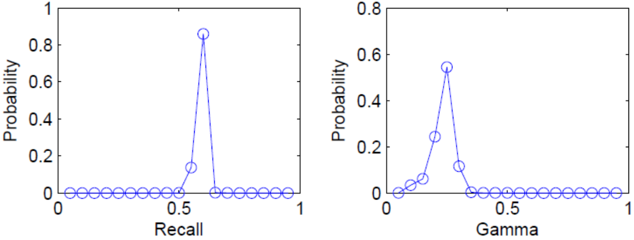
- The risk aversion hierarchical parameter distribution is located below .50 implying the mean of the risk aversion parameter equals .65, indicating somewhat strong risk aversion
- The loss aversion hierarchical parameter distribution is located above .50 implying the mean of the loss aversion equals 1.97, higher sensitivity to losses

HBA estimates of group level parameters



- The hierarchical memory parameter is slightly above .50 implying the mean of the memory parameter equals .59

HBA estimates of group level parameters



- The hierarchical memory parameter is slightly above .50 implying the mean of the memory parameter equals .59
- The hierarchical distribution for the key quantum parameter lies below .50 implying a mean value equal to -2.67.

Concluding Comments

- HBA analysis produced posterior for gamma < 0 (quantum regime)

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- HBA analysis produced posterior for gamma < 0 (quantum regime)
- Too early to conclude the quantum model is always superior to a Markov model, need to examine more data sets
- Also need to examine more prior distributions

Further Reading

Thank you

- Busemeyer, J. R. & Bruza, P. D. (2012, June) Quantum models of cognition and decision. Cambridge University Press.