Adaptive dynamics and its application to context dependent systems breaking the classical probability law

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Adaptive Dynamics

Existence of substance, more generally, of any observable depends on how it is observed. How can we describe this fact (the existence itself) mathematically? What is philosophical and mathematical basis describing the appearance of existence? It is the adaptive dynamics (AD) to answer these questions.

AD is a mathematical method describing the appearance of existence (subsistent) in a specific angle.

⟺

AD is the theory of existence how it exists under various effects surrounding it.

In fact, **AD is a detailed mathematics to describe the existence (subsistent) by taking various effects surrounding it.**

Adaptive dynamics

The adaptive dynamics(AD) has two aspects, one of which is the "observable-adaptive" and the other is the "state-adaptive“[3,5].

The **observable-adaptive dynamics** is a dynamics characterized as follows:
(1) Measurement depends on how to see an observable to be measured.
(2) The interaction between two systems depends on how a fixed observable exists.

The **state-adaptive dynamics** is a dynamics characterized as follows:
(1) Measurement depends on how the state to be used exists.
(2) The correlation between two systems interaction depends on how the state of at least one of the systems at one instant exists.

⇨ **Mathematical expression is given in the sequel sections**

Applications

1. Description of chaos: Introduce a chaos degree
2. Solve the millennium problem “NPC=P”
3. Give a new proof of the Schor’s factorizing algorithm.
4. Study a bio-system and a psycho-system; the evolution of HIV-1, the brain function and the irrational behavior of prisoners.
5. **Find a mathematics to calculate the joint and conditional probabilities**

   ⇒ New probability law (Non-Kolmogorovian) ⇒ We explain this today.
6. Our new formulation is discussed in the book (**“Mathematical Foundations of Quantum Information and Computation and Its Applications to Nano- and Bio-systems”, Springer-Verlag 2011)**.

On quantum algorithm

(1)Apply the AD to solve the problem NPC=P in two ways ; using Chaos amplification (MO and Volovich) and Weak coupling limit (Accardi and MO).

(2) **Recent Results**: We pointed out some incompleteness in the proof of Shor's quantum factoring algorithm[10]. In these papers, the followings are not clear:

1. <u>**How to construct his unitary operator.**</u>
2. <u>**How to erase the dust qubits to achieve Shor's reduction.**</u>

-> We proposed the other quantum factoring algorithm [12] by using a new quantum search algorithm , which can avoid the above difficulties.

Let f be a function from {0,1,⋯,2^n−1} to {0,1}, we consider the following searching problem:

Problem(Searching problem) Find x such that $f(x)=1$.

(2) We developed a quantum algorithm for searching problem based on OV SAT algorithm, and its computational complexity T is **polynomial in** log$N$ **(c.f. Grover's search is √N)[11,12]** where N is a number of elements in the problem.

Theorem 2 We have

$$T = \frac{13}{8}n^2 - \frac{9}{4}n + nT\left(U_f\right)$$

where n= log$N$, and $T(Uf)$ is a given complexity associated to the function f.

[10] S.Iriyama, M.Ohya, I.V.Volovich, On Computational Complexity of Shor's Quantum Factoring Algorithm, TUS preprint, (2012)

[11] S.Iriyama, M.Ohya, I.V.Volovich, On Quantum Algorithm for Binary Search and Its Computational Complexity, TUS preprint, (2012)

[12] K.Goto, S.Iriyama, M.Ohya, I.V.Volovich, On Quantum Algorithm of Prime Factoring Using Quantum Binary Search, in preparation

New Probability Law

There exist several phenomena (systems) breaking the usual probability laws.

- Quantum interference
- Various biological systems

- State change of tongue for sweetness[3]
- Bayesian updating biased by Psychological factor[3,8] ⇒ **Khrennikov's talk**
- Lactose-glucose interference in E. coli growth [3,4] ⇒ **Tanaka's talk**

These phenomena (systems) will require us a change of usual probability law.

It is important to notice that these phenomena are context dependent, so that they are **adaptive to the context of the surroundings**.

In this study, **we apply the concept of the adaptive dynamics** to make a mathematical framework for the study of these context dependent systems.

- In context dependent systems, the usual probability theory and Bayesian statistics are broken (or should be modified)!
- For instance, $P(A|B) \neq P(A,B)/P(B)$

$$\Rightarrow P(A) \neq \sum_i P(A|B_i)P(B_i)$$

- We have to make a new probability law!
- Non-commutative theory is needed even for classical objects!

Reference:

“[6] Mathematical Foundations of Quantum Information and Computation and Its Applications to

Nano- and Bio-systems”, Springer-Verlag 2011.

Lifting

In order to partially solve the difficulty of the nonexistence of joint quantum distribution, the notion of compound state satisfying the marginal condition is useful. In this section we discuss a bit general notion named "**lifting**"[7](Accardi and MO) to discuss new scheme of probability containing both classical and quantum.

Definition Let be C^*-algebras and let be a fixed C^*-tensor product $\mathcal{A}_1$ and $\mathcal{A}_2$. A lifting from $\mathcal{A}_1$ to $\mathcal{A}_1 \otimes \mathcal{A}_2$ is a weak $*$ -contiouous map

$$\mathcal{E}^* : \mathcal{S}(\mathcal{A}_1) \to \mathcal{S}(\mathcal{A}_1 \otimes \mathcal{A}_2)$$

If is affine and its dual is a completely positve map, we call it a linear lifting; if it maps pure states into pure states, we call it pure.

Definition A lifting $\mathcal{E}^*$ from $\mathcal{A}_1$ to $\mathcal{A}_1 \otimes \mathcal{A}_2$ is called nondemolition for a state $\varphi_1 \in \mathcal{S}(\mathcal{A}_1)$ if φ_1 is invariant for a channel Λ^* i.e., if for all $A_1 \in \mathcal{A}_1$

$$\Lambda^*\varphi_1 \equiv \left(\mathcal{E}^*\varphi_1\right)\left(A_1 \otimes 1\right) = \varphi_1\left(A_1\right)$$

The idea of this definition being that the interaction with system 2 does not alter the state of system 1.

$$\mathcal{E}^*\text{: lifting} \quad \Leftrightarrow \quad \Lambda^*\text{: channel } \mathcal{S}(\mathcal{A}_1) \to \mathcal{S}(\mathcal{A}_1)$$

Example Quantum measurement: If a measuring appratus is prepared by an positive operator valued measure $\{Q_n\}$ then the state ρ changes to a state $\Lambda^*\rho$ after this measurement,

$$\rho \to \Lambda^*\rho = \sum_n Q_n \rho Q_n.$$

Here

$$\mathcal{A}_1 = \mathbf{B}(\mathcal{H}_1), \quad \mathcal{A}_2 = \mathbb{C}.$$

Example Reduction (Open system dynamics): If a system Σ_1 interacts with an external system Σ_2 described by another Hilbert space $\mathcal{K}$ and the initial states of Σ_1 and Σ_2 are ρ_1 and ρ_2, respectively, then the combined state θ_t of Σ_1 and Σ_2 at time t after the interaction between two systems is given by

$$\theta_t \equiv U_t \left(\rho_1 \otimes \rho_2\right) U_t^* = \mathcal{E}^* (\rho_1)$$

where $U_t = \exp(-itH)$ with the total Hamiltonian H of Σ_1 and Σ_2. A channel is obtained by taking the partial trace w.r.t. $\mathcal{K}$ such as

$$\rho_1 \rightarrow \Lambda^* \rho_1 \equiv \mathrm{tr}_{\mathcal{K}} \mathcal{E}^* (\rho_1).$$

Example Let $\Lambda^* : \mathcal{S}(\mathcal{A}_1) \to \mathcal{S}(\mathcal{A}_2)$ be a channel. For any $\rho_1 \in \mathcal{S}(\mathcal{A}_1)$ in the closed convex hull of the external states, fix a decomposition of ρ_1 as a convex combination of extremal states in $\mathcal{S}(\mathcal{A}_1)$

$$\rho_1 = \int_{\mathcal{S}(\mathcal{A}_1)} \omega_1 d\mu$$

where μ is a Borel measure on $\mathcal{S}(\mathcal{A}_1)$ with support in the extremal states, and define

$$\mathcal{E}^* \rho_1 = \int_{\mathcal{S}(\mathcal{A}_1)} \omega_1 \otimes \Lambda^* \omega_1 d\mu$$

Then $\mathcal{E}^* : \mathcal{S}(\mathcal{A}_1) \to \mathcal{S}(\mathcal{A}_1 \otimes \mathcal{A}_2)$ is a lifting, nonlinear even if Λ^* is linear, and it is a nondemolition type.

$\Rightarrow$ This type of lifting can be applied to study the quantum mutual entropy and the “entanglement”.

Let us show some important examples of liftings and channels below.

Exapmle5 Let $V:\mathcal{H}_1 \to \mathcal{H}_1 \otimes \mathcal{H}_2$ be an isometry

$$V^*V = 1_{\mathcal{H}_1}$$

Then the map

$$\mathcal{E}: x \in \mathbf{B}(\mathcal{H}_1) \otimes \mathbf{B}(\mathcal{H}_2) \to V^* x V \in \mathbf{B}(\mathcal{H}_1)$$

is a transition expectation in the sense of Accardi, and the associated lifting maps a density matrix w_1 in $\mathcal{H}_1$ into

$$\mathcal{E}^* w_1 = V^* w_1 V$$

in $\mathcal{H}_1 \otimes \mathcal{H}_2$. Liftings of this type are called isometric. Every isometric lifting is a pure lifting. In this case the channel $\Lambda^*:\mathcal{H}_1 \to \mathcal{H}_1$ is given by $\mathrm{tr}_{\mathcal{H}_2} \mathcal{E}^*$.

New view of probability theory

The adaptive dynamics is considered that the dynamics of a state or an observable after an instant (say the time t_0) attached to a system of interest is affected by the existence of some other observable and state at that instant.

$\rho \in \mathcal{S}(\mathcal{H})$ $\cdots$a state before $t_0 (=0)$ $\mathcal{S}(\mathcal{H})$ $\cdots$ set of density operators

$\left(A \in \mathcal{A} \right.$ $\cdots$an observable before $\left. t_0 \right)$

$\sigma \in \mathcal{S}(\mathcal{H} \otimes \mathcal{K})$ $\cdots$a state to give an effect to ρ and A $\Leftrightarrow$ state-adaptive

$Q \in \mathcal{A} \otimes \mathcal{B}$ $\cdots$an observable to give an effect to ρ and A

$\Leftrightarrow$ observable-adaptive

$\mathcal{E}_{\sigma Q}^{*}$ $\cdots$a lifting describing the whole adaptivity to ρ

> The adaptive dynamics is the process such as
>
> **Adaptive Dynamics**: $\rho \Rightarrow \mathcal{E}_{\sigma Q}^{*} \rho \Rightarrow \rho_{\sigma Q} = \mathrm{tr}_{\mathcal{K}} \mathcal{E}_{\sigma Q}^{*} \rho$

What we need is how to construct the lifting for each problem to be studied.

Now suppose that there are two general event systems

$$A = \{a_k \in \mathbb{R}, E_k \in \mathcal{A}\} \text{ and } B = \{b_j \in \mathbb{R}, F_j \in \mathcal{B}\}$$

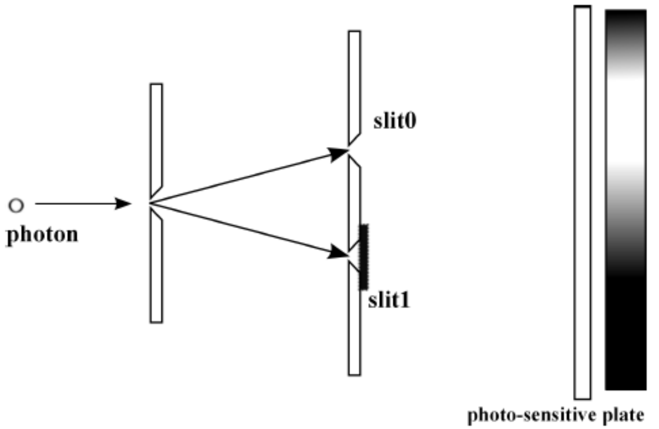
where we do not assume E_j, F_k are projections but they satisfy the conditions $\sum_j E_j = I, \sum_k F_k = I$ as POVM (positive operator valued measure) corresponding to the partition of a probability space in classical system. Then the "joint-like" probability obtainning a_k and b_j might be given by

$$P(a_k, b_j) = \mathrm{tr} E_k \boxdot F_j \mathcal{E}^*_{\sigma Q} \rho,$$

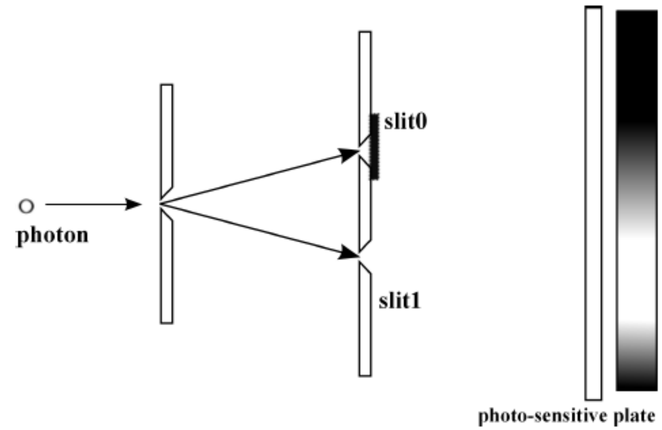
where $\boxdot$ is a certain operation (relation) between A and B, more generally one can take a certain operator function $f(E_k, F_j)$ instead of $E_k \boxdot F_j$. (*e.g.* tensor product $\otimes$ in a simple case)

New mathematical law computing the probability for double-slit experiment

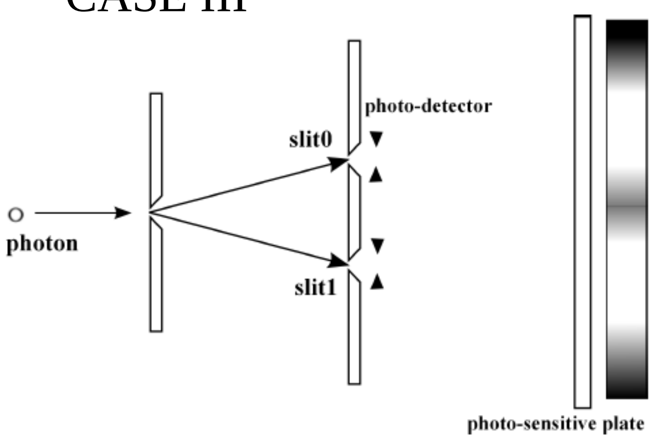
CASE I



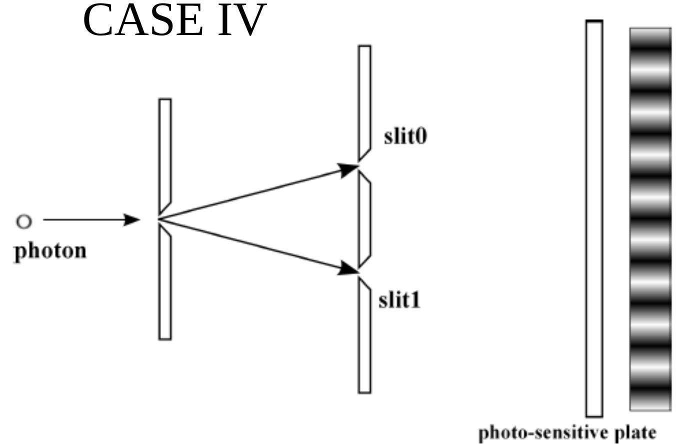
CASE II



CASE III



CASE IV



$$\rho = |\phi\rangle\langle\phi| \qquad |\phi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$\mathcal{E}_Q^* : \mathcal{H}_{slit} = \mathbb{C}^2 \mapsto \mathcal{H}_{slit} \otimes \mathcal{H}_{plate} = \mathbb{C}^2 \otimes L^2(\mathbb{C})$$

$$\mathcal{E}_Q^*(\rho) = \rho \otimes \frac{Q\rho Q^*}{\mathrm{tr}(QQ^*\rho)}$$

$$Q = |\psi_0\rangle\langle 0| + |\psi_1\rangle\langle 1|$$

$$|\psi_i\rangle = \int \psi_i(x)|x\rangle dx$$

$$\rho_{plate} = \mathrm{tr}_{\mathcal{H}_{slit}}(\mathcal{E}_Q^*(\rho))$$

$$P(x) = \mathrm{tr}(|x\rangle\langle x|\rho_{plate}) = \frac{|\alpha\psi_0(x) + \beta\psi_1(x)|^2}{1 + \alpha\beta(\langle\psi_0|\psi_1\rangle + \langle\psi_1|\psi_0\rangle)}$$

$$P(x) = \mathrm{tr}(|x\rangle\langle x|\,\rho_{plate}) = \frac{|\alpha\psi_0(x) + \beta\psi_1(x)|^2}{1 + \alpha\beta(\langle\psi_0|\psi_1\rangle + \langle\psi_1|\psi_0\rangle)}$$

$\alpha = 1$ $\quad P(x) = |\psi_0(x)|^2$ CASE I

$\beta = 1$ $\quad P(x) = |\psi_1(x)|^2$ CASE II

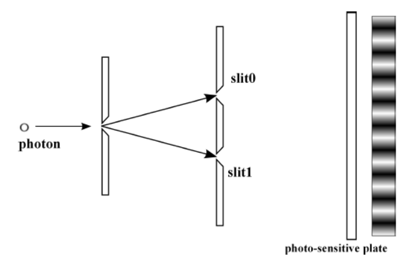
If $\langle\psi_0, \psi_1\rangle = 0$

$$P(x) = |\alpha\psi_0(x) + \beta\psi_1(x)|^2$$

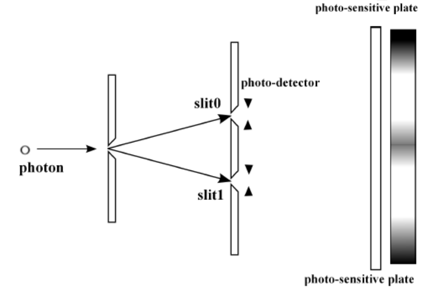
Standard form

$$\neq |\alpha|^2|\psi_0(x)|^2 + |\beta|^2|\psi_1(x)|^2$$

CASE IV



CASE III



Violation of total probability law in bio-systems

There are many experimental data breaking usual probability law such as in Ecoli's behavior, Brain function, Optical illusion

Example: State change of tongue for sweetness

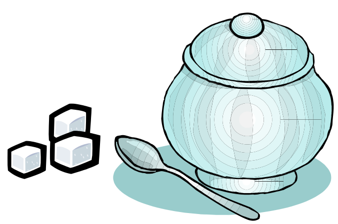
One takes sugar S and chocolate C

and he tastes 1(sweet) , 0(not sweet).

Then the classical probability law is not satisfied:

$$P(C=1) \neq P(C=1|S=1\ )\ P(S=1) + P(C=1|S=0)\ P(S=0)$$

because LHS is very close to 1 and RHS will be less than 0.5.



$\mathcal{H} = \mathcal{K} = \mathbb{C}^2$

$\mathcal{A}, \mathcal{B}$: sets of all observables on Hilbert spaces $\mathcal{H}, \mathcal{K}$, resp.

$\mathcal{S}(\mathcal{H}), \mathcal{S}(\mathcal{K})$: sets of all states on Hilbert spaces $\mathcal{H}, \mathcal{K}$, resp.

General event system1

$\{a_k \in \mathbb{R}, E_k \in \mathcal{A}\}$ $\quad E_k = |e_k\rangle\langle e_k|$ $\quad \{|e_k\rangle\}$: CONS of $\mathcal{H}$

General event system2

$\{b_k \in \mathbb{R}, F_k \in \mathcal{B}\}$ $\quad F_k = |f_k\rangle\langle f_k|$ $\quad \{|f_k\rangle\}$: CONS of $\mathcal{K}$

<Event system1>

$S = 1$: Sugar is sweet $\leftrightarrow E_1 = |e_1\rangle\langle e_1|$

$S = 0$: Sugar is not sweet $\leftrightarrow E_0 = |e_0\rangle\langle e_0|$

<Event system2>

$C = 1$: Chocolate is sweet $\leftrightarrow F_1 = |f_1\rangle\langle f_1|$

$C = 0$: Chocolate is not sweet $\leftrightarrow F_0 = |f_0\rangle\langle f_0|$

New mathematical law computing the probability in adaptive dynamics for sweetness

Initial state $\rho = |x\rangle\langle x|$ $\quad |x\rangle = \frac{1}{\sqrt{2}}|e_0\rangle + \frac{1}{\sqrt{2}}|e_1\rangle \quad |e_0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, |e_1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Lifting

$$\mathcal{E}^*_{\sigma,Q} : \mathcal{S}(\mathcal{H}) \to \mathcal{S}(\mathcal{H} \otimes \mathcal{K})$$

$$\mathcal{E}^*_{\sigma,Q}\rho \left(= \mathcal{E}^*_{S,X,C}\rho \right) \equiv \rho_S \otimes \rho^a_{S\to C}$$

Adaptive change

$$\rho_S \equiv \frac{S\rho S^*}{\mathrm{tr}(|S|^2 \rho)} \in \mathcal{S}(\mathcal{H}) \qquad \rho^a_S \equiv X\rho_S X \qquad \rho^a_{S\to C} = \frac{C\rho^a_S C^*}{\mathrm{tr}(|C|^2 \rho^a_S)} \in \mathcal{S}(\mathcal{K})$$

$$S = \begin{pmatrix} \lambda_0 & 0 \\ 0 & \lambda_1 \end{pmatrix} \qquad X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad C = \begin{pmatrix} \mu_0 & 0 \\ 0 & \mu_1 \end{pmatrix}$$

$$|\lambda_1|^2 + |\lambda_0|^2 = 1 \qquad |\mu_1|^2 + |\mu_0|^2 = 1$$

Common experience tells us that $|\lambda_1|^2 > |\mu_1|^2 \gg |\mu_0|^2 > |\lambda_0|^2$

Formula computing Joint probabilities

$$P(S=j, C=k) = \mathrm{tr}\left[\left(E_j \otimes F_k\right)\mathcal{E}^*_{S,X,C}\rho\right]$$

$$P(S=1,C=1)+P(S=0,C=1)=\mathrm{tr}_{\mathcal{H}\otimes\mathcal{K}}\left[\left(I\otimes F_1\right)\mathcal{E}^*_{S,C}\rho\right]=\frac{|\mu_1|^2|\lambda_0|^2}{|\mu_1|^2|\lambda_0|^2+|\mu_0|^2|\lambda_1|^2}$$

Note that this probability is much less than

$$|\lambda_1|^2 > |\mu_1|^2 \gg |\mu_0|^2 > |\lambda_0|^2$$

$$P\left(C=1\right) \equiv \mathrm{tr}_{\mathcal{K}} F_1 \Lambda^*_C \rho = |\mu_1|^2 \qquad \Lambda^*_C \equiv \mathrm{tr}_{\mathcal{H}} \mathcal{E}^*_{S,C}\rho$$

With these probabilities, the TPL is violated.

$$P\left(C=1\right) \neq P(S=1,C=1)+P(S=0,C=1)$$

⇒ More realistic study is the behavior of Escherichia coli. **(Tanaka's talk)**

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